Let S be a sphere rotating at constant angular velocity about an axis which passes through its center. Let C be a great circle on S. As p varies over all points on C, the magnitude of the projection onto the plane of C of the rotational velocity vector at p is constant.

Proof: Without loss of generality, we assume that the sphere has radius 1 and that the angular velocity is 1 radian per second. We choose a coordinate system with origin at the center of the sphere and with $x-$, $y-$, and $z-$axes chosen so that the plane of C is given by $z = 0$.

Choose orthonormal vectors v_i, $i = 1, 2, 3$ so that v_3 is the axis of rotation and so that the rotation carries the positive v_1 towards the positive v_2. Let the coordinates of $v_i = (x_i, y_i, z_i)$ for $i = 1, 2, 3$. Let $P = av_1 + bv_2 + cv_3$ be an arbitrary point on C, which means that we have $az_1 + bz_2 + cz_3 = 0$ and $a^2 + b^2 + c^2 =$1. The rotational velocity vector at P is then $-bv_1 + av_2$ and its projection onto the plane of C is $(-bx_1 + ax_2, -by_1 + ay_2, 0)$. The square of the length of this projection is given by

$$(-bx_1 + ax_2)^2 + (-by_1 + ay_2)^2 =$$

$$a^2 \left(x_2^2 + y_2^2\right) + b^2 \left(x_1^2 + y_1^2\right) - 2ab \left(x_1 x_2 + y_1 y_2\right) =$$

$$a^2 \left(1 - z_2^2\right) + b^2 \left(1 - z_1^2\right) - 2ab \left(v_1 \cdot v_2 - z_1 z_2\right) =$$

$$a^2 \left(z_1^2 + z_3^2\right) + b^2 \left(z_2^2 + z_3^2\right) + 2ab z_1 z_2$$

because v_1, v_2, , v_3 are orthonormal. Regrouping,

$$= (az_1 + bz_2)^2 + a^2 z_3^2 + b^2 z_3^2$$

$$= (-cz_3)^2 + a^2 z_3^2 + b^2 z_3^2$$

$$(c^2 + a^2 + b^2) z_3^2 = z_3^2,$$

which does not depend on the point P. To express this invariantly, the length of this projection is the length of the projection of the unit direction of the axis of rotation upon the direction orthogonal to that of the great circle.