

The computation on our page 65 (the top of page 94 in the French edition) of how far off-center the cueball may be struck without miscuing. This distance is given by that a satisfying the relationship

$$\frac{5}{2} \frac{a^2}{R^2} = \left(1 + \frac{M'}{M}\right) + \left\{ -\frac{\theta}{2} \left(1 + \frac{M'}{M}\right) \sqrt{\frac{\theta^2}{4} \left(1 + \frac{M'}{M}\right)^2 + 1 - \theta \left(1 + \frac{M'}{M}\right)} \right\}$$

This indeed follows from the equation

$$\left(1 + \frac{M'}{M}\right) \sqrt{1 - \theta - \theta \left(\frac{M'}{M}\right) \left(1 + \frac{5}{2} \frac{a^2}{R^2}\right)} = \frac{5}{2} \frac{a^2}{R^2}$$

which is given as an inequality just before the previous equation above is asserted. I lost my way several times verifying this, so I thought that I'd write it up.

To see this, let $X = 1 + \frac{5}{2} \frac{a^2}{R^2}$ and let $Q = \frac{M'}{M}$. Then, if we substitute in the latter equation and square both sides, we get

$$(1 + Q^{-1})^2 (1 - \theta - \theta Qx) = (x - 1)^2 = x^2 - 2x + 1, \text{ or}$$

$$x^2 + [\theta Q (1 + Q^{-1})^2 - 2] x + [1 - (1 + Q^{-1})^2 (1 - \theta)] = 0$$

Applying the quadratic formula we have

$$x = -\frac{1}{2} [\theta Q (1 + Q^{-1})^2 - 2] \pm \frac{1}{2} \sqrt{\theta^2 Q^2 (1 + Q^{-1})^4 - 4\theta Q (1 + Q^{-1})^2 + 4 - 4(1 + Q^{-1})^2 (1 - \theta)},$$

or

$$x = -\frac{1}{2} [\theta Q (1 + Q^{-1})^2 - 2] \pm \frac{(1 + Q^{-1})}{2} \sqrt{\theta^2 Q^2 (1 + Q^{-1})^2 - 4\theta Q + -4(1 - \theta)}$$

or

$$x - 1 = (1 + Q^{-1}) \left[-\frac{\theta}{2} Q (1 + Q^{-1}) + \sqrt{\frac{\theta^2}{4} (Q + 1)^2 - \theta Q + 1} \right],$$

which becomes precisely the formula near the bottom of our page 65 (or on the top of page 94 in the original.)