We recall that in the case of a horizontal stroke, as we assume here, we have

$$\frac{a^2}{R^2} = \frac{h^2}{R^2} + \left(1 - \frac{l}{R}\right)^2 .$$

In order to examine the maximum values of y_0 and y_r, it will at once be necessary to set $h = 0$, because, given the smallness of θ, the value of h will have hardly any influence except in the denominator. Let us assume therefore that $h = 0$— that is, let us consider only the case where the vertical plane of impact passes through the center of the ball. Let us substitute for $\frac{a^2}{R^2}$ its value in this case, which is $\left(1 - \frac{l}{R}\right)^2$; we then obtain

$$y_0 = \frac{W'^2 2}{2fg} \frac{\left(1 - \frac{l^2}{R^2}\right)\left[1 + \sqrt{1 - \theta - \theta\frac{M'}{M}\left(1 + \frac{5}{2}\left(1 - \frac{l}{R}\right)^2\right)}\right]^2}{\left(1 + \frac{M}{M'} + \frac{5}{2}\left(1 - \frac{l}{R}\right)^2\right)^2}$$

and

$$y_r = \frac{W'^2}{2fg} \frac{\left(1 - \left(\frac{5}{7}\frac{l}{R}\right)^2\right)\left[1 + \sqrt{1 - \theta - \theta\frac{M'}{M}\left(1 + \frac{5}{2}\left(1 - \frac{l}{R}\right)^2\right)}\right]^2}{\left(1 + \frac{M}{M'} + \frac{5}{2}\left(1 - \frac{l}{R}\right)^2\right)^2}$$

We can introduce into these formulas the velocity that the cueball would have assumed with the same stroke of the cue if it had been cued center-ball: we then have

$$W_0 = W' \frac{1 + \sqrt{1 - \theta - \frac{\theta M'}{M}}}{1 + \frac{M}{M'}},$$

whence[10]

$$W' = \frac{W_0\left(1 + \frac{M}{M'}\right)}{1 + \sqrt{1 - \theta - \frac{\theta M'}{M}}}$$

Substituting this value in the preceding expressions, we have[11]

$$y_0 = \frac{W_0^2}{2fg} \frac{\left(1 - \frac{l^2}{R^2}\right)\left(1 + \frac{M}{M'}\right)^2\left[1 + \sqrt{1 - \theta - \frac{M'\theta}{M}\left(1 + \frac{5}{2}\left(1 - \frac{l}{R}\right)^2\right)}\right]^2}{\left(1 + \sqrt{1 - \theta - \frac{\theta M'}{M}}\right)^2\left(1 + \frac{M}{M'} + \frac{5}{2}\left(1 - \frac{l}{R}\right)^2\right)^2}$$